# Stability of symmetric and antisymmetric solitons in $PT$-symmetric couplers

Rodislav Driben and Boris A. Malomed

Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel e-mail: malomed@post.tau.ac.il

**Abstract:**

Dual-core *couplers*, i.e., systems of parallel waveguides (*cores*) linearly coupled by tunneling of light between them, play a fundamental role in optics, as well as in other physical applications. In case the cores carry the intrinsic Kerr nonlinearity, the coupler gives rise to (temporal or spatial) solitons, which remain symmetric at small values of the energy (power), and undergo spontaneous symmetric breaking, i.e., the destabilization of symmetric solitons and a transition to stable asymmetric ones, at a critical value of energy. In fact, the spontaneous symmetry breaking of solitons in couplers is one of basic manifestations of this effect, which occurs in a broad variety of nonlinear symmetric systems [1].

Recently, a model of a $\mathcal{PT}$-symmetric nonlinear coupler, with linear gain ($\gamma$) applied to one core, and the balancing loss acting in the mate one, was proposed [2,3]. In the general form, this model is based on the following system of propagation equations for amplitudes $\psi_{1,2}$ of electromagnetic waves in the linearly coupled cores, written in terms of the spatial-domain propagation:

$$i\left(\psi_1\right)_z + \left(\psi_1\right)_{xx} + \left(|\psi_1|^2 + \chi|\psi_2|^2\right)\psi_1 + \psi_2 = i\gamma\psi_1, \quad (1)$$

$$i\left(\psi_2\right)_z + \left(\psi_2\right)_{xx} + \left(\chi|\psi_1|^2 + |\psi_2|^2\right)\psi_2 + \psi_1 = -i\gamma\psi_2, \quad (2)$$

where the linear-coupling coefficient is scaled to be 1, and real coefficient $\chi$ accounts for the possible nonlinear coupling between the cores. Symmetric and antisymmetric soliton solutions of this system are available in an analytical form [4]. The stability boundary for the symmetric solitons against symmetry-breaking perturbations in the case of $\chi = 0$ was recently found in an exact form [1,2]. We here report exact results for the stability boundary of symmetric and antisymmetric solitons in the general model (1),(2). The main finding is that the stability region for the squared amplitude of the solitons, known in an analytical form in the conservative model ($\gamma = 0$), is reduced by factor $\sqrt{1-\gamma^2}$. The stability region shrinks to nil at $\gamma = 1$, when the the gain/loss coefficient is equal to the constant of the inter-core linear coupling, making the system "supersymmetric" [5] at this point.

**References:**

1. B. A. Malomed (editor), *Spontaneous Symmetry Breaking, Self-trapping, and Josephson Oscillations* (Springer, 2013).
2. R. Driben and B. A. Malomed, Opt. Lett. 36, 4323 (2011).
3. N. V. Alexeeva, I. V. Barashenkov, A. A. Sukhorukov, and Yu. S. Kivshar, Phys. Rev. A 85, 063837 (2012).
4. H. H. Sakaguchi and B. A. Malomed, Phys. Rev. E 83, 036608 (2011).
5. R. Driben and B. A. Malomed EPL 96, 51001 (2011).